Optimization-Based Formation Control of Underactuated Magnetic Microrobots via Inter-Agent Forces

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***Abstract*—This paper presents a new optimization-based method to control three micro-scale magnetic agents operating in close proximity to each other for applications in microrobotics. Controlling multiple magnetic microrobots close to each other is difficult due to magnetic interactions between the agents, and here we seek to control those interactions for the creation of desired multi-agent formations. Our control strategy arises from physics that apply force in the negative direction of states errors. The objective is to regulate the inter-agent spacing, heading and position of the set of agents, for motion in two dimensions, while the system is inherently underactuated. Simulation results on three agents and a proof-of-concept experiment on two agents show the feasibility of the idea to shed light on future micro/nanoscale multi-agent explorations. Average tracking error of less than 50 micrometers and 1.85 degrees is accomplished for the regulation of the inter-agent space and the pair heading angle, respectively, for identical spherical-shape agents with nominal radius less than of 250 micrometers operating within several body-lengths of each other.**

I. Introduction

Magnetic actuation of microrobots has been recognized as a safe and efficient approach to access small remote spaces with a wide range of potential applications in drug delivery [1], cell lysis/sorting [2], micro-assembly/disassembly [3]. The ability to exert independent control over a team of microrobots working together on a task has potential to increase task speed and capability to perform parallel operations [4]. However, team control of magnetic micro-agents remains an open-ended problem as in most actuation systems, all magnetic micro-agents share a global driving magnetic signal. In this way, all agents receive identical control inputs and thus it is difficult to steer independently for complex task completion [5].

In the microrobotics field, a variety of approaches have been explored toward the team control of magnetic microrobots: Miyashita et al. [6] utilized local magnetic interaction forces to create a few stable formations in two dimensions of a set by dynamically remagnetizing some of the agents. However, that method is limited to a small set of stable configurations, has no control over the formation orientation, cannot be generalized to microrobots moving in three dimensions, and is only applicable to sets of agents which are each magnetically unique. Cappelleri et al. [7] utilized row-column planar addressing micro-coils made with printed circuit board (PCB) technology to localize the field driving magnetic robots. Mellal et al. [8] incorporated optimal linear quadratic integrator (LQI) control to navigate two magnetic microbeads independently in 1D. Nevertheless, all these studies have been done only for a small number of agents or entangled with severe limitations. Recently, Abbot et al. [9] derived a general compact analytic approach using linear-algebraic representations to find a minimum-power dipole solution with application to electromagnetic formation flight. However, their approach may not be applicable to microrobotic system that is heavily underactuated and the relation between coil currents and inter-dipole forces is not always linear.

Our previous work [10] for the first time introduced a method to control the motion of two identical agents in close proximity with one control input angle only, while magnetic moment was constrained to the horizontal plane. Additionally, the full 2D motion control of the two-agent configuration was accomplished in [11] with two control input angles, while magnetic moment was free to rotate in 3D. To that end, the spatially-uniform external field created only torque to orient micro-agents and as a result the inter-agent force appeared between agents. The agents' magnetic moment angles were modulated to regulate this force. Here, we apply the same general use of inter-agent forces, but with a new controller for three identical agents to coordinate their relative positions and orientation in close proximity. However, the system is now underactuated as there are only two control input angles associated to the orientation of the magnetic moments versus at least three states required to describe the relative position of the three agents. Since there is no exact solution to this underactuated system, this work employs a fitness function optimization method. The proposed approach can pave the path for the team control of microrobots at micro/nanoscales.

The paper is structured as follows. Section II describes the

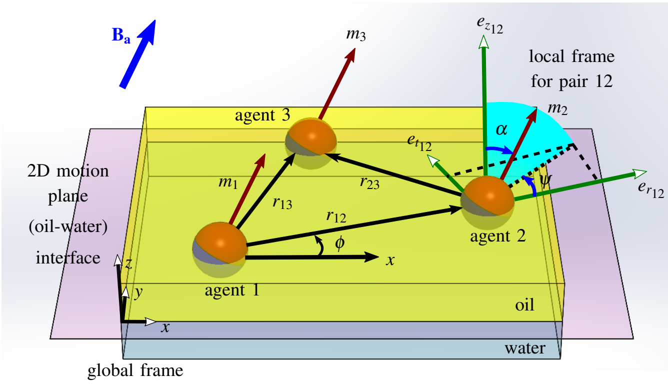

Fig. 1: 3D pairwise orientation parameters defined in global and local coordinates for agents in close proximity with magnetic moments $\boldsymbol{m}_1$, $\boldsymbol{m}_2$ and $\boldsymbol{m}_3$ aligned with the actuation field $\boldsymbol{B}_a$. The pairwise distance vectors connecting agent i to agent j are denoted by $\boldsymbol{r}_{ij}$, where $i, j \in \{1,\ 2,\ 3\}$. The radial and transverse coordinates are assigned locally with respect to each pair. The actuation field and magnetic moments' orientation are all defined arbitrarily with respect to the first pair axis (team's local reference) shown by $e_{r_{12}}$ and $e_{t_{12}}$. The local coordinates associated to other pairs are not sketched. The 2D motion of the agents occurs at the interface between water and oil. The out-of-plane angle that magnetic moments m makes with the z-axis within the cyan plane is denoted by α and the in-plane angle that the projection of magnetic moment makes with the reference radial axis in the motion plane is shown by ψ. 3D total forces acting on each agent are not shown for simplicity but clearly explained in the text.

kinematics of agents and our method to regulate the relative motion of three agents. Section III presents our simulation and experimental results. This paper is concluded in section IV.

II. Control of three-agent configuration

This section introduces the kinematics describing generic representation of a team of three agents, and lays the foundation for controlling a three-agent configuration.

A. Magnetic actuation and inter-agent kinematics

Following the convention, magnetic flux density is denoted by $\boldsymbol{B}$. A magnetic moment $\boldsymbol{m}$ represents the field orientation of a magnetic microrobot agent. Under the act of an external magnetic field or via local magnetic interaction with other agents of a set, each agent may experience both force $\boldsymbol{F}_m$ and torque $\boldsymbol{\tau}$, which can be calculated by $\boldsymbol{F}_m = (\boldsymbol{m} \cdot \nabla)\boldsymbol{B}$ and $\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{B}$, where ∇ here is the material gradient [12]. We base our analysis on the assumption that the magnetic moment $\boldsymbol{m}$ of all magnetic agents in the workspace align with the applied field $\boldsymbol{B}_a$. We use the angle(s) of the applied field as our control input(s) to the entire system. Consider three identical magnetic agents with magnetic moments $\boldsymbol{m}$ aligned with the uniform external magnetic field $\boldsymbol{B}_a$ applied in the workspace for actuation in a two-dimensional (2D) horizontal plane at oil-water interface as sketched in Fig. 1. As the applied field is uniform over space, no external magnetic forces are generated ($\nabla \boldsymbol{B}_a = 0$).

The local in-plane control input angle ψ defined as the angle between the projection of the actuation field $\boldsymbol{B}_a$ and the vector $\boldsymbol{r}_{12}$ in the motion plane as sketched in Fig. 1. Similarly, $\psi_G = \psi + \phi$ is the in-plane control angle in global coordinates xyz. The out-of-plane (tilting) control input angle measured down from the z-axis is denoted by α. In this work agents are capable to magnetically rotate in 3D. In other words, both out-of-plane and in-plane rotations are feasible. To realize this assumption, we use spherical agents with minimal surface area to demand least agent-liquid surface energy. As such, magnetic moment orientation can be characterized freely in 3D by 2 DOF variables ψ and α. We express all vectors with respect to a global coordinate frame after all. Let ${}^{\mathrm{i}}\boldsymbol{F}_{\mathrm{tot}}$ denote the total force vector created at the location of agent i by the rest of agents of the set, then ${}^{\mathrm{i}}\boldsymbol{F}_{\mathrm{tot}} = \sum_{k \neq i} \boldsymbol{F}_{ki}$ whereby $k \in \{1,\ 2,\ \ldots,\ n\}$ with n as the number of agents, and

$$\boldsymbol{F}_{ki}(r_{ki},\ \psi,\ \alpha) = \tfrac{3\mu_0}{4\pi r_{ki}{}^5}[(\boldsymbol{m}_k \cdot \boldsymbol{r}_{ki})\boldsymbol{m}_i + (\boldsymbol{m}_i \cdot \boldsymbol{r}_{ki})\boldsymbol{m}_k + (\boldsymbol{m}_k \cdot \boldsymbol{m}_i)\boldsymbol{r}_{ki} - \tfrac{5(\boldsymbol{m}_k \cdot \boldsymbol{r}_{ki})(\boldsymbol{m}_i \cdot \boldsymbol{r}_{ki})}{r_{ki}^2}\boldsymbol{r}_{ki}] \tag{1}$$

is the pairwise magnetic force exerted at the location of agent i by agent k [12]. Here μ_0 is the permeability of free space, $\boldsymbol{m}_k = \boldsymbol{m}_i$ is the magnetic moment vector, $\boldsymbol{r}_{ki}$ is the separation vector connecting agent k to agent i, and r_{ki} is the norm of this vector. In local Cartesian coordinates defined exclusively for each pair of agents $(\hat{e}_{r_{ij}}, \hat{e}_{t_{ij}}, \hat{e}_{z_{ij}})$, the net radial and transverse components of the total magnetic force exerted on agent j by the rest of agents, linked to pair ij can be written as

$$ {}^{\mathrm{j}}\boldsymbol{F}_{r_{ij}} = ({}^{\mathrm{j}}\boldsymbol{F}_p \cdot \frac{\boldsymbol{r}_{ij}}{\|\boldsymbol{r}_{ij}\|})\hat{e}_{r_{ij}}, \text{ and} \tag{2a}$$

$$ {}^{\mathrm{j}}\boldsymbol{F}_{t_{ij}} = (\text{z-component}\{{}^{\mathrm{j}}\boldsymbol{F}_p \times \frac{\boldsymbol{r}_{ij}}{\|\boldsymbol{r}_{ij}\|}\})\hat{e}_{t_{ij}}. \tag{2b}$$

where ${}^{\mathrm{j}}\boldsymbol{F}_p$ is the projection of ${}^{\mathrm{j}}\boldsymbol{F}_{\mathrm{tot}}$ in the motion plane. Without loss of generality, we make this assumption that magnetic tilting force ${}^{\mathrm{j}}\boldsymbol{F}_{z_{ij}}$ will be counteracted by the surface tension at the liquid interface. To obtain the states velocities, the subtraction of total forces after being projected to the associated pair separation vector by a dot or cross product is calculated depending whether the state is of radial (r_{ij}) or transverse (ϕ_{ij}) form, and represent these difference terms for agent i by ${}^{\mathrm{i}}F_{r_{ij}}$ and ${}^{\mathrm{i}}F_{t_{ij}}$, respectively.

$$\dot{r}_{ij} = \frac{{}^{\mathrm{ij}}F_r}{\sigma} = \frac{{}^{\mathrm{i}}F_{r_{ij}} - {}^{\mathrm{j}}F_{r_{ij}}}{\sigma}, \text{ and} \tag{3a}$$

$$r_{ij}\dot{\phi}_{ij} = \frac{{}^{\mathrm{ij}}F_t}{\sigma} = \frac{{}^{\mathrm{i}}F_{t_{ij}} - {}^{\mathrm{j}}F_{t_{ij}}}{\sigma}. \tag{3b}$$

Here σ is the fluid drag constant. The microrobots used in this study are spheres with the nominal radius of 250 μm, and are experiencing low Reynolds number laminar fluid flow with a negligible net acceleration (inertia) on each agent. Therefore, a first-order model is considered to describe the agents' motion based on the Stokes fluid drag model.

B. Optimization-based control law

The state representation of three agents is not unique. In general, a three-agent configuration can be represented by a triangle as shown in Fig. 1. To fully describe the system globally in 2D, there are four states needed. For example, three separations and one pair heading $\boldsymbol{x} = [r_{12}\ r_{13}\ r_{23}\ \phi]^T$, or two separations with the intermediate angle plus over one pair heading $\boldsymbol{x} = [r_{12}\ r_{13}\ \theta_1\ \phi]^T$ whereby ϕ can be the heading of any arbitrary pair. Our goal is to find ψ and α angles solution that minimizes a weighted L^2-norm fitness function so that the relative spacings and angles of the pairs are pushed toward the desired ones between a set of 3 magnetic agents. Let's state our optimization problem as follows:

$$\begin{aligned}\underset{\psi,\ \alpha}{\text{minimize}} \quad f = & \sum_{i,j=1}^{n} \|\varphi(e_{r_{ij}}(t+1|t)) + \gamma\ {}^{\text{ij}}F_r\|^2 \\ & + \lambda\Big(\sum_{i,j=1}^{n} \|\varphi(e_{t_{ij}}(t+1|t)) + \gamma\ {}^{\text{ij}}F_t\|^2\Big) \|D(\frac{e_{t_{ij}}}{\Gamma})\|,\end{aligned} \tag{4}$$

The proposed fitness function is a weighted sum of corrective radial forces and transverse forces. To distinguish the opposite sign of the radial and transverse forces φ is considered which could be either a binary function such as $\varphi = \text{sgn}(.)$ leading to a binary response around the desired states, or a smooth logistic function such as $\varphi = 2(\frac{1}{1+e^{-\beta x}} - 0.5)$, whereby the error input is denoted by x and the slope of attraction and repulsion toward the goal is denoted by β. Also λ denotes the weight to specify the tracking of whichever state of separation r or pair heading angle ϕ is more important for a particular trajectory. For two-agent configuration, the error in transverse state ϕ denoted by e_t gets zero at $\psi = 0°$ or $90°$. However, these angles may spontaneously generate undesirable largest radial forces that would lead to a small steady error in separation state (refer to [11] for further details). One can simply compensate this small artifact by defining a deadzone $D(.)$ over the transverse state with a small width Γ with value around $0.5°$.

III. Results

A. Simulations

A simulation environment based on the proposed physical model is designed to enable prediction of agents' behaviors under the act of controller so as to be able to justify and optimize the experimental observations. Fig. 2 shows numerical simulations of the motion trajectory of magnetic agents in three-agent configuration using optimization-based controller. This set of simulations are a time integration of agents' position vector velocity, $\dot{\boldsymbol{x}}_i = -\frac{{}^{\text{i}}\boldsymbol{F}_{\text{tot}}}{\sigma}$, $i \in \{1,\ 2,\ 3\}$ for various initial conditions of triangle and collinear, including parameter $|m| = 10^{-6}$ Am^2. Due to the non-convexity of fitness function expressed in (4), 4-start gradient descent optimization was employed to guarantee finding the global minimum. The agent-to-agent capillary force and agent-liquid-wall capillary force, as well as inertial forces are ignored in the simulation. Here we only discuss three candidate scenarios of three-agent configurations.

a) Symmetrical collinear case: In Fig. 2(a), a symmetrical collinear case is investigated where three agents are initially too far from each other and positioned symmetrically in a line with respect to the third agent at center. The controller task is to choose the input magnetic field angle(s) to push the relative spacing and pair heading angle toward the goal state: $(r_{12} = 2r_{\text{des}}, r_{13} = r_{23} = r_{\text{des}}, \phi_{12} = \phi_{\text{des}})$. The desired pairwise separation r_{des} is reached when the sketched surrounding dashed-line circles around agents with radius equal to $0.5r_{\text{des}}$ come into contact with one another. Initial positions are denoted by circle and current positions with diamond. The desired separation and pair heading are set at $7R$ and $45°$, respectively. It can be seen from Figs. 2(a) and 2(b) that the controller approaches the goal configuration and the error reduces to a small value over time. We have seen in simulation that the controllers are stable for the symmetrical collinear case for a wide variety of initial conditions. The switching behavior of the control input is apparent both in the trajectory inset and in the time-series graph in Fig. 2(b) to maintain the pairs' orientation at desired angle. It can be noted that the symmetrical collinear is an augmented case of two-agent configuration with the third agent staying stationary.

b) Equilateral triangle case: Fig. 2(c) shows the case where three agents initially hold a random triangle configuration. The controller task is to choose the input magnetic field angle(s) to push the relative spacings only toward an equilateral triangle goal state: $(r_{12} = r_{13} = r_{23} = r_{\text{des}})$. It is clear from Figs. 2(c) and 2(d) that the controller approaches the goal configuration and the error reduces to a small value over time. We have seen in simulation that equilateral triangle goal is reachable for some of initial conditions.

c) Trapping linear case: Fig. 2(e) shows the case where three agents start from a random triangle configuration and are forced to reach an equilateral triangle. However, agents get stuck at a line configuration appeared on their planned path. The special hint about linear configuration is that no transverse forces can be created at the corresponding solution angles leading the agents to get stuck in a line. Further details can be found at [11]. It can be justified from the graphs in Fig. 2(f) that once the line trapping happens, control angles ψ and α aggressively starts switching back and forth as highlighted in the yellow window. The reason is that in our current tests, the multi-start optimization routine always strictly returns the lowest global minima out of multiple existing global minima. Therefore, for the sake of continuity this consideration should be applied into our future attempts that whenever algorithm faces with multiple global minima at the current time step, it chooses whichever minimum that is closest to the solution from the previous time step.

We note that the total separation error $e_{r_{tot}}$ in the first two cases is monotonically decreasing until it reduces to zero at goal state: a good empirical sign that the proposed control law is effective for these configurations.

B. Experimental setup

Our identical smooth spherical agents are produced in a batch process using a fluid-assisted method similar to what we established in [11]. Magnetic fields for agent actuation are created in an electromagnetic coil system with three pairs

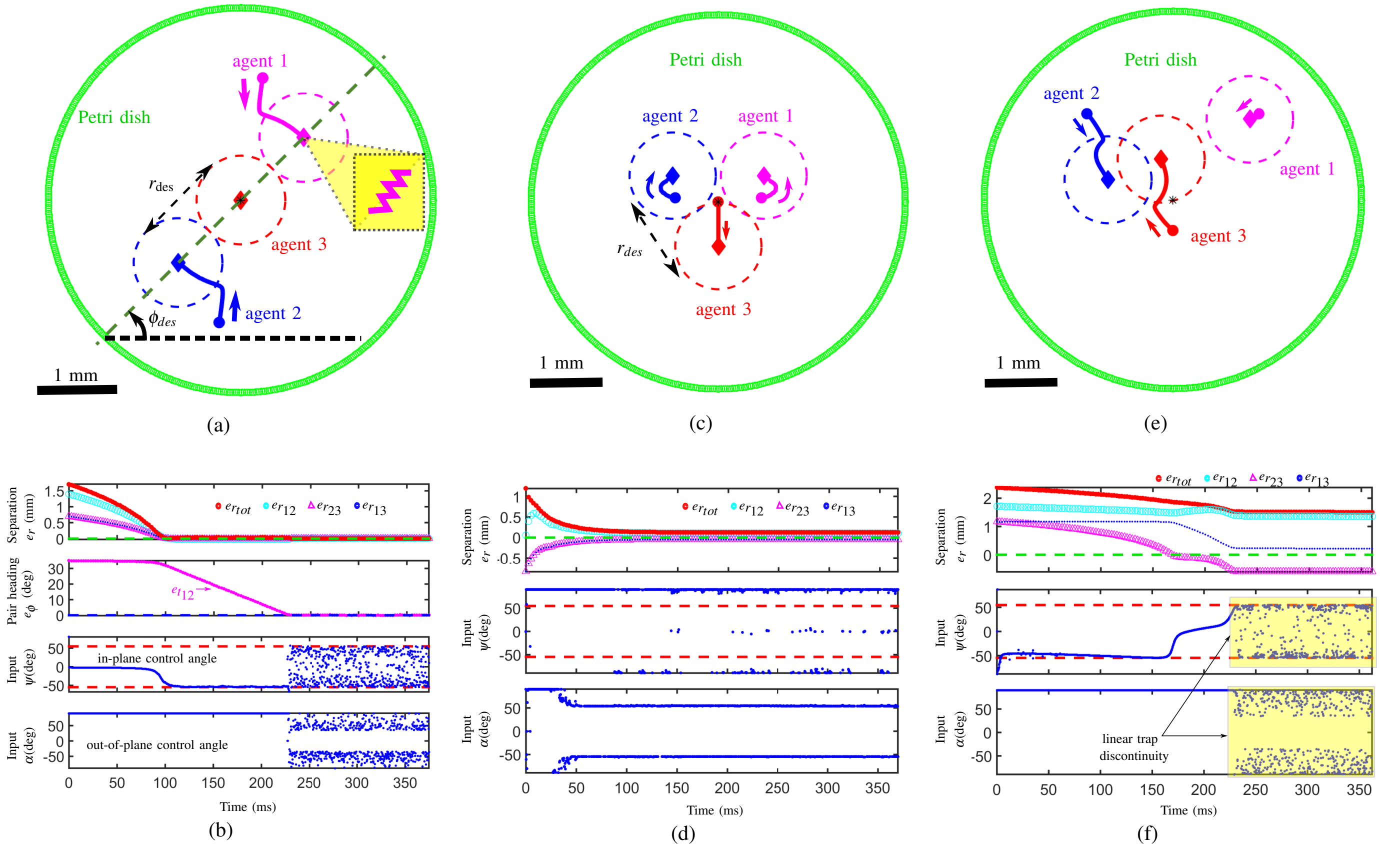

Fig. 2: Three-agent configuration control simulations using multi-start gradient descent method. From left to right are shown the motion trajectory and time evolution of separations and pair heading angle states errors along with the control inputs ψ and α for: (left) symmetrical collinear, (middle) equilateral triangle, and (right) trapping linear configurations. The top plots show the motion trajectory simulations in solid line. The desired pairwise separation r_{des} is reached when the sketched surrounding dashed-line circles around agents with radius equal to $0.5r_{\text{des}}$ come into contact with one another. Initial positions are denoted by circle and current positions with diamond. Here the agent radius is 250 μm, the desired separation and pair heading are set at $7R$ and $45°$, respectively, and $|m| = 10^{-6}$ Am^2. Pairwise separation errors and total L^2-norm of these errors are represented by $e_{r_{ij}}$ and $e_{r_{tot}}$, respectively.

of coils nested orthogonally to create fields in 3D. Further details on electronic setup and image detection can be found at [11]. As we need to deal with multiple identical agents in this work, a Kalman filter was implemented to associate the detections corresponding to the same agent over time. Two identical agents are immersed in a glass Petri dish and sit at water-oil interface as illustrated in Fig. 3.

C. Experiment

Two-agent configuration is a building block of three agents. Therefore, as a preliminary proof-of-concept of three-agent configurations, we tested this new optimization-based method on two-agent configuration to evaluate the performance of the controller. Fig. 4 shows the experimental result for the controller to track a changing goal state. RMS tracking error of less than 50 μm and 1.85° is accomplished for the regulation of the separation r and the pair heading angle ϕ, respectively. Thus, the optimization-based controller has the capability to operate as efficient as our previous controllers in [11]. The control inputs are bounded and following a trend under the influence of the controller. Also one can control the center-of-mass (COM) position of the set of agents besides

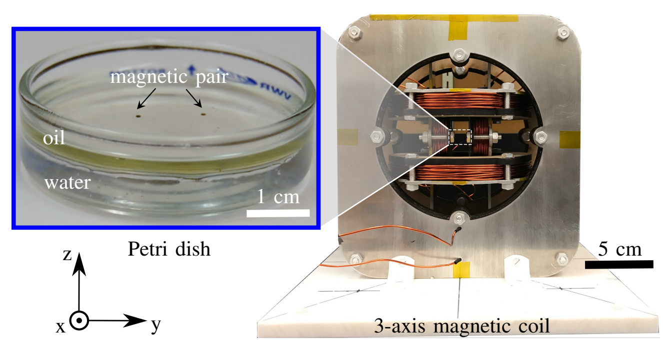

Fig. 3: Experimental setup. In the inset image of agent, two spherical microrobots sit at the interface of water-oil inside a glass Petri dish. The agents are driven in horizontal plane by an electromagnetic coil system with three pairs of coils capable of producing fields in 3D.

the relative states using a 2D magnetic field gradient that is superimposed on the uniform field signal. Further details can be found at [11]. A video of this experiment is available as supplementary material [S1].

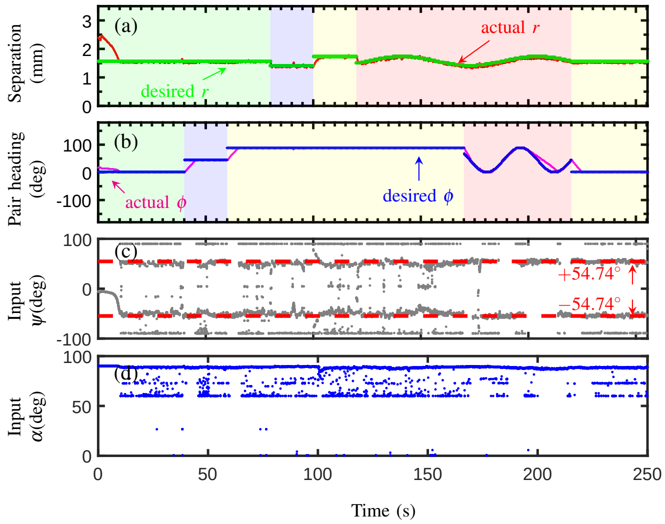

Fig. 4: (a) Separation r and (b) pair heading ϕ tracking using optimization-based control at water-oil interface, (c) the in-plane control angle ψ and (d) the out-of-plane tilting control angle α. RMS tracking error of less than 50 μm and 1.85° is accomplished for the regulation of the separation and the pair heading angle, respectively. A video of this experiment is available as supplementary material [S1].

IV. Conclusion and Discussions

In this paper, we proposed a new systematic controller to tune the 2D motion of three identical magnetic microrobots in close proximity with each other using the same framework we developed in the past with focus on the inter-agent forces. We conducted numerical simulations to verify the method over two particular configurations of symmetrical collinear and equilateral triangle. Furthermore, some challenges were identified in terms of optimization routine that need to overcome in future studies. Proof-of-concept test performed on two agents shows evidence of effectiveness of the proposed controller to be considered as a tool for running the three-agent experiments. The presented controller in this paper was just a preliminary approach to systematically formulate the problem of multi-agent formation control of magnetic microrobots and not the most elegant one. In future work, we will investigate the use of an underactuated technique and stability analysis to make the proposed controller generalized for three or more agents. For example, instead of reaching the goal by taking a path direction for which input forces do not exist one can decompose that direction into multiple basis directions for which input forces do exist.

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